HOW NOISY SHOULD A NOISE SIGNAL BE? OPTIMAL LEVEL OF NOISE FOR BANK STABILITY AND DEPOSITOR WELFARE

Geethanjali Selvaretnam
University of St Andrews, UK[1]
E-mail: gs51@st-andrews.ac.uk

ABSTRACT

There is a strand of literature on bank runs, where depositors decide whether or not to withdraw their deposits based on noisy signals about the viability of the bank. The models used in these papers assume that the level of noise is very small and go on to establish a unique equilibrium with a threshold level below which depositors would withdraw. Noise indicates the level of transparency of the bank's future financial state. In reality however, noise need not be very small. The level of transparency of the information that is made available to the depositors can be endogenised so that it is chosen by the banks or the regulators. This paper attempts to determine the optimal level of noise for bank stability and depositor welfare. The objective of the financial regulators and the authorities would be to minimise the probability of bank runs, while the objective of banks operating in a competitive environment would be to maximise the expected utility of depositors. This paper uses a simple theoretical model of bank runs to demonstrate that there should be high level of transparency about the banks' future profitability to both minimise bank runs and maximise the expected utility of depositors.

JEL Classifications: C72, D82, G21, G28

Keywords: bank runs, transparency, noisy signal

---

[1] Address: School of Economics and Finance, University of St Andrews, 1 The Scores, St Andrews KY16 9AL; Email: gs51@st-andrews.ac.uk; Tel: 0044(0)1334 461956; Fax: +44(0)1334 462444.

1. INTRODUCTION

The literature on bank runs has gone through various developments over the last three decades. Diamond and Dybvig (1983) showed how bank runs are caused by self-fulfilling beliefs of depositors. Their model has two equilibria: the bank run equilibrium and the no bank run equilibrium. A crucial break through was made to establish unique equilibrium applying the global game framework introduced by Carlsson and van Damme (1993) where agents receive noisy signals about the fundamental. In such bank run models, depositors would withdraw if the signal is below a threshold point (Morris and Shin (2002), Goldstein and Pauzner (2004, 2005), Dasgupta (2004)). The fundamental gives information about the long term earning potential of the bank. Using the noisy signal, each depositor can work out the long term return that can be expected. These models assume near-precise information about the fundamental with the range of noise close to zero. The noise reveals how informative the signal is about the true value of the fundamental. Lower the noise, higher the level of informativeness and transparency of the signal. In reality, it is possible to choose the amount of information that is made available to the agents. How transparent a bank should be regarding its future returns, is an important policy decision. For empirical evidence that shows that transparency increases financial stability, see Erland (2005) and Nier (2005).

This paper provides a simple and formal model to analyse how transparent the information should be - in other words, how noisy should a noisy signal be. This aim is to find the optimal level of transparency that should be chosen for stability in the banking industry and depositor welfare. We can allow those in charge, such as the managers of banks, or the regulators of banks to decide how much information is to be made available to depositors. First, the model analyses the case where the decision is made by the authorities who want to minimize the probability of bank runs. The next section determines the level of noise that maximizes the expected utility of the depositors, which would be the objective of banks which operate in competitive markets. It is found that for both objectives, noise should be very small (i.e. high level of transparency). If the noise is very large, it means that the private information is of no value and therefore the depositors don't act on it. If there is to be information, it is better to make it as transparent as possible.

The rest of the paper is organized as follows: In the next section, the model is set up. In Section 3 the probability of bank runs is minimized while in Section 4 the expected utility of the agents is maximized. Section 5 concludes.

2. THE MODEL

The basic framework follows Goldstein and Pauzner (2005). There are three periods (t_0, t_1, t_2). There is a continuum $[0,1]$ of agents who are the depositors, and one bank operating in a competitive environment. Each agent is endowed with one unit at the beginning of t_0, which is invested in the bank. Consumption happens only in periods t_1 and t_2. All agents are identical and risk averse, and each agent's utility function is strictly concave, increasing and twice continuously differentiable.

A fraction λ of the agents are hit by a liquidity shock in t_1, which requires them to definitely withdraw early. If the agents withdraw in t_1 they will receive an early return of 1, the same amount that they deposited. The bank keeps λ as reserves to meet the demand of impatient agents who are hit by the liquidity shock and invests the balance $(1-\lambda)$ in a long term project. Each unit that is invested till t_2 will realize a random return θ.

The depositors might also want to withdraw early because they believe that if they don't, they might end up losing their investment in the bank because sufficiently large number of agents decides to withdraw early. If the patient agents (those who are not hit by the liquidity shock) want to withdraw early in t_1, the bank borrows from an outside party to meet the demand. Because the loan will be from an institution which has the welfare of the financial system in consideration, it is assumed that the bank will be able to borrow whatever that is needed to meet the early demand. This loan has to be settled with interest, so that each unit that is borrowed will have to be repaid with $L(\geq 1)$. Those who do not withdraw in t_1 will have an equal share of whatever remains of the earnings from the long term project after the loan is settled (if there is anything left to share) in t_2.

The economic fundamental θ is uncertain and is drawn from a uniform distribution on $[\underline{\theta}, \theta]$ where $\underline{\theta} = 0$ and $\overline{\theta}$ very large. In t_1 each agent i observes a noisy signal $\theta_i = \theta + \varepsilon_i$ of the economic fundamental θ. The noise ε_i is uniformly and independently distributed among the depositors with support $(-e, +e)$. Once the agents observe the signal, they will decide whether to withdraw in t_1 or wait till t_2. This decision is based on their beliefs about θ and the number of agents who would withdraw in t_1. A threshold θ^* can be established where a player withdraws if and only if he observes a θ_i less than θ^* in t_1.

The crucial factor in this model is that the range of the noise level, e, is a choice variable. Lower the noise, e, higher the transparency / information that is available to the agents about their long term return.

It is assumed that the fundamental θ has an upper dominant region and lower dominant region. This assumption is required for a unique equilibrium to be established, as is explained in the literature on global games. If θ was such that no patient agent withdraws, the return to the agent by not withdrawing is θ. If θ is sufficiently low such that $u(\theta) < u(1)$, it is better to withdraw early even if no other agent withdraws. If player i's signal is $\theta_i < 1 - e$, he will definitely withdraw.

On the other hand there could be a range of θ which is so high that even if everyone else withdraws it is better for an agent not to withdraw. If everyone else withdraws, by waiting he will receive $u(\theta - L)$. If $u(\theta - L) > u(1)$ the agent is better off by not withdrawing, which means that if the signal he receives is $\theta_i > 1 + L + e$ he will definitely not withdraw. As is customary in this literature, when computing θ^* we only consider the range $[\theta^* - e, \theta^* + e]$ and assume that the dominant regions are extreme enough that they will not have an influence over θ^*. This is particularly true because when noise is large, depositors will not consider their signals at all because the signals will not be informative.

2.1. Threshold level $\boldsymbol{\theta^*}$

We can compute a threshold level of θ^* where if any agent observes a value less than θ^* in t_1, he will withdraw. Once the economic fundamental θ is realized, each player i receives a signal $\theta_i = \theta + \varepsilon_i$. Symmetric threshold strategy would mean ${\theta_i}^* = \theta^*$ for every player$i$. If $\theta_i > \theta^*$, agent i believes that the bank's investment is doing sufficiently well and large enough proportion of the depositors believe the same. Therefore he would not withdraw. If $\theta_i < \theta^*$, agent i believes that sufficiently high proportion of depositors believe (as he does) that they should withdraw because the bank will not be sufficiently profitable.

Proposition 1: *There exists a threshold level of the fundamental θ^* such that a patient agent will withdraw if and only if he observes a signal less than θ^*.*

Proof: Let the agents withdraw early if they receive a signal less than θ.

Player i who observes signal θ_i has a posterior distribution of θ that is given by $y(=\theta/\theta_i)$. We know that y is then uniformly distributed on $[\theta_i - e, \theta_i + e]$ where each of the points is realized with equal probability $\frac{1}{2e}$. In turn he will believe that each of the point $y \in [\theta_i - e, \theta_i + e]$ would have given out signals to the other agents $(y - e, y + e)$ meaning the proportion of patient agents who he believes would withdraw would be a distribution given by $\omega(y) \in [0,1]$:

$$\left.\begin{aligned} y &\leq \theta - e, & \omega &= 1. \\ \theta - e < y &< \theta + e, & \omega &= \frac{\theta - \ y - e}{2e}. \\ y &\geq \theta + e, & \omega &= 0. \end{aligned}\right\} \qquad 1$$

If a patient agent who observes $\theta_i = \theta^*$ withdraws in t_1 he will definitely receive one unit. If he does not run, he will receive $max \ \frac{y-\omega L}{1-\omega}, 0$ because the agent will never receive a negative return.

If $y < \omega L \ (i.e. y < \frac{(\theta+e)L}{L+2e}$, he believes he will receive nothing in the last period.

If $y > \omega L \ (i.e. y > \frac{(\theta+e)L}{L+2e}$, he believes he will receive $\frac{y-\omega L}{1-\omega}$ in the last period.

The difference in expected utility from withdrawing and not withdrawing, given signal θ_i is given by:

$$g = EU(withdraw/_{\theta_i}) - EU(not\ withdraw/_{\theta_i}). \qquad (2)$$

$$g = u \ 1 \ - \ _{\frac{\theta+e \ L}{L+2e}}^{\theta_i+e} \ \frac{1}{2e} u \ \frac{y - \omega \ y \ \ * L}{1 - \omega \ y} \quad dy. \qquad (3)$$

$$g: R \rightarrow R.$$

$$\lim_{\theta_i \rightarrow \underline{\theta}} g > 0 \ \ and \ \ \lim_{\theta_i \rightarrow \overline{\theta}} g < 0.$$

Over the range of the integral in (3), $u(.)$ is non negative.

$$\frac{dg}{d\theta_i} = -\frac{1}{2e} u \quad \frac{\theta_i \ + \ e \ - \ \frac{\theta \ - \ \theta_i}{2e} \ L}{1 - \frac{\theta - \theta_i}{2e}} \quad . \qquad (4)$$

Because $g(.)$ is continuous in θ_i and decreasing we can conclude that there exists a unique point where $g(\theta^*) = 0$, so that the agent who receives a signal $\theta_i = \theta^*$ will be indifferent between withdrawing and not withdrawing early.

3. MINIMIZING THE PROBABILITY OF BANK RUNS

In this section we focus on the results which minimize the probability of bank runs. The objective of the authorities who aim to enhance stability in the banking industries would be to minimize the probability of bank runs. If they have a hand in determining the level of noise and the interest that has to be paid to the lenders, it is reasonable to assume that they would choose e and L to minimize θ^*.

According to Proposition 2, transparency reduces the probability of bank runs. When there is nearly full transparency and information provided to agents is easily interpretable $(e \to 0)$, the probability of bank run is minimized. This result supports the empirical evidence provided by Erland (2005) and Nier (2005) whose papers show that transparency increases financial stability.

If noise is very small, the signal each agent receives is very close to the true value. However, if the authorities are unable to ensure much transparency (or if the depositors are not sophisticated enough to make good use of the information that is provided), should we have large noise? If the noise is very large, depositors can't learn anything from their private signals and therefore would not consider it when making decisions. Because it is difficult to prevent some information floating around, this model recommends that there should be very clear transparency of information (i.e. very small level of noise) if probability of bank runs is to be minimized. In other words, information should be provided in such a way that agents can predict their long term return accurately.

> **Proposition 2:** *The probability of a bank-run is minimized when the noise level, e is at a minimum.*

Proof: We use equation (5) which gives the indifference condition where the agent is indifferent between withdrawing and not withdrawing when he receives a signal $\theta_i = \theta^*$.

$$h(\theta^*, e, L) := \int_{\frac{(\theta^*+e)L}{L+2e}}^{\theta^*+e} \frac{1}{2e} u\left(\frac{y-\omega(y)*L}{1-\omega(y)}\right) dy - 1. \qquad (5)$$

$$\frac{\partial h}{\partial e} = \frac{2e * u(\theta^* + e) - 2\int_{\frac{(\theta^*+e)L}{L+2e}}^{\theta^*+e} u\left(\frac{y-\omega(y)*L}{1-\omega(y)}\right) dy}{4e^2} \qquad (6)$$

When $y = \theta^* + e,\ u\left(\frac{y-\omega(y)*L}{1-\omega(y)}\right) = u(\theta^* + e)$.

We can depict this diagrammatically in Figure 1.

$$e * u(\theta^* + e) = BCDE, \qquad (7)$$

$$\frac{1}{2e}\int_{\frac{(\theta^*+e)L}{L+2e}}^{\theta^*+e} u\left(\frac{y-\omega(y)*L}{1-\omega(y)}\right) dy = ACD. \qquad (8)$$

If e is small enough so that $BCDE < ACD$ (i.e. $DEF < ABF$).

This means that

$$e * u(\theta^* + e) < \frac{1}{2e}\int_{\frac{(\theta^*+e)L}{L+2e}}^{\theta^*+e} u\left(\frac{y-\omega(y)*L}{1-\omega(y)}\right) dy.$$

So, from (6) we can say that $\frac{\partial h}{\partial e} < 0$. We also know that $\frac{\partial h}{\partial \theta^*} > 0$.

$$\frac{d\theta^*}{de} = -\frac{\frac{\partial h}{\partial e}}{\frac{\partial h}{\partial \theta^*}} > 0. \qquad (9)$$

Therefore we can conclude that $\frac{d\theta^*}{de} > 0$, i.e. to minimize θ^*, noise level e should be at a minimum.

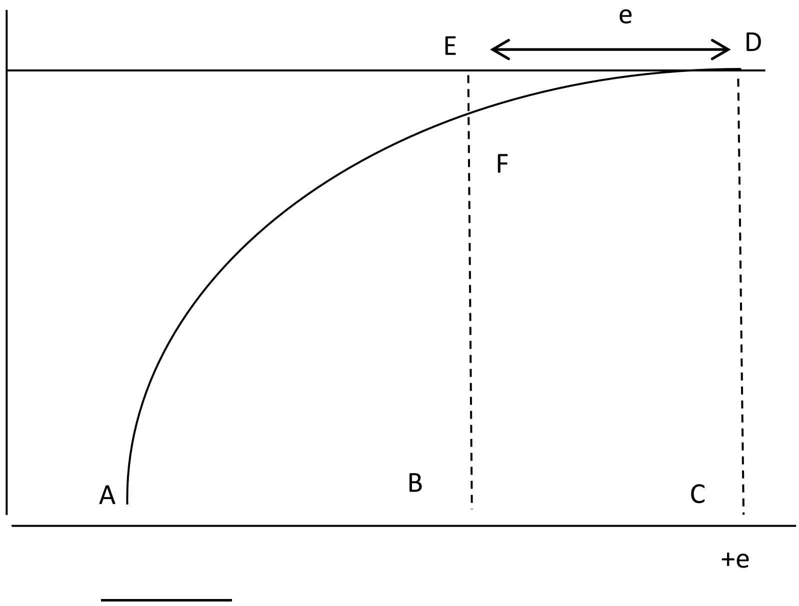

Figure 1: Utility of expected long term return

4. MAXIMIZING DEPOSITOR WELFARE

In this section we look at the outcome when the objective is to maximize the expected utility of the depositors. This would be the objective of a bank which operates in a competitive market, when it makes a decision about what information to divulge. The expected utility of a representative depositor is given by (10).

$$EU = \frac{1}{\overline{\theta} - \underline{\theta}} \left[\lambda u(1) + (1-\lambda) \left[\int_{\underline{\theta}}^{\theta^*-e} u(1)\, d\theta + \int_{\theta^*+e}^{\overline{\theta}} u\left(\frac{\theta}{1-\lambda}\right) d\theta + \int_{\theta^*-e}^{\theta^*+e} \omega(\theta) * u(1)\, d\theta + \int_{\theta^*-e}^{\frac{(\theta^*+e)L}{2e+L}} (1-\omega(\theta)) * 0\, d\theta + \int_{\frac{(\theta^*+e)L}{2e+L}}^{\theta^*+e} (1-\omega(\theta)) * u\left(\frac{\theta-\omega(\theta) * L}{(1-\lambda)(1-\omega(\theta))}\right) d\theta \right] \right] \tag{10}$$

The probability of θ being at any point is *ex ant* $\frac{1}{\overline{\theta}-\underline{\theta}}$. With λ probability, the agent can be hit by the liquidity shock and have to withdraw early. In which case, his utility would be $u(1)$. With $(1-\lambda)$ probability, he will not be hit by the liquidity shock. In which case, his expected utility is given within the square brackets. The first term is when $\theta < \theta^* - e$, so that all the depositors would withdraw early in t_1 and receive early return of 1 for sure. The second term is when $\theta > \theta^* + e$, so that all the patient depositors would wait till t_2 and therefore the entire earning, θ, will be distributed among the patient depositors.

When θ is between $\theta^* - e$ and $\theta^* + e$ we have a partial run where the agent might have to either run or not run. With $\omega(\theta)$ probability an agent would withdraw and with $(1 - \omega(\theta))$ probability he would not withdraw. If he does withdraw, he will receive utility of one unit. But if he does not run, he could either receive nothing (if too many depositors had withdrawn early) or $u \ \frac{\theta - \omega L}{(1-\omega)(1-\lambda)}$.

The proportion of agents who run can be looked at in three categories.

$\omega = 0$ if $\theta > \theta^* + e$. When θ is large enough, no patient agent will run.

$\omega = 1$ if $\theta < \theta^* - e$ When θ is low enough everyone will run.

$\omega = \frac{\theta^* - \theta + e}{2e}$ if $\theta^* - e \leq \theta \leq \theta^* + e$. This is when there will be a partial run.

According to Proposition 3 the expected utility of the agents is maximized if information is as close to the true value as possible.

Proposition 3: *To maximize the expected utility of the depositors, the level of noise e, should be minimized.*

Proof: The expected utility of an agent given in equation (10) can be rearranged as follows in (11).

$$\begin{aligned} EU = & \frac{\lambda}{\overline{\theta}-\underline{\theta}} \Bigg[u(1) + \int_{\underline{\theta}}^{\theta^*-e} u(1)\, d\theta + \int_{\theta^*+e}^{\overline{\theta}} u\left(\frac{\theta}{1-\lambda}\right) d\theta + \int_{\theta^*-e}^{\theta^*+e} \omega(\theta) * u(1)\, d\theta \Bigg] \\ & + \frac{1-\lambda}{\overline{\theta}-\underline{\theta}} \Bigg[\int_{\frac{\theta^*+e\,L}{2e+L}}^{\theta^*+e} u\left(\frac{\theta-\omega(\theta) * L}{(1-\lambda)(1-\omega(\theta))}\right) d\theta + \int_{\frac{\theta^*+e\,L}{2e+L}}^{\theta^*+e} \omega(\theta)\, u\left(\frac{\theta-\omega(\theta) * L}{(1-\lambda)(1-\omega(\theta))}\right) d\theta \Bigg] \end{aligned} \tag{11}$$

Keep in mind the following:

$$\frac{d\omega}{de} = \frac{\theta-\theta^*}{2e^2}; \tag{12}$$

$$\frac{d}{de} u\left(\frac{\theta-\omega(\theta) * L}{(1-\lambda)(1-\omega(\theta))}\right) = \frac{u'(.)\left[(1-\omega)\left(-L\frac{d\omega}{de}\right) + (\theta-\omega * L)\frac{d\omega}{de}\right]}{(1-\lambda)(1-\omega)^2}$$

$$= \frac{u'(.)\left(\frac{\theta-\theta^*}{2e^2}\right)(\theta-\mathrm{L})}{(1-\lambda)(1-\omega)^2}; \tag{13}$$

$$\frac{d}{de}\omega u \left(\frac{\theta - \omega L}{(1-\lambda)(1-\omega)}\right) = u \,.\, \left(\frac{\theta - \theta^*}{2e^2}\right) + \omega \frac{u' \,.\, \left(\frac{\theta-\theta^*}{2e^2}\right) L(\theta - 1)}{(1-\lambda)(1-\omega)^2} \quad (14)$$

$\omega = 0$ when $\theta = \theta^* + e$; $\omega = 1$ when $\theta = \theta^* - e$; $\omega = \frac{\theta^*+e}{2e+L}$ when $\theta = \frac{(\theta^*+e)L}{2e+L}$.

$$\frac{dEU}{de} = \frac{1}{\bar{\theta} - \underline{\theta}} \int_{\frac{(\theta^*+e)L}{2e+L}}^{\theta^*+e} \left(\frac{\theta - \theta^*}{2e^2}\right)\left[\frac{u' \,.\, (\theta - L)}{(1-\lambda)(1-\omega)} - u\left(\frac{\theta - \omega L}{(1-\lambda)(1-\omega)}\right)\right] d\theta \quad (15)$$

Because $u(c) > cu'(c)$,

$$u\left(\frac{\theta - \omega L}{(1-\lambda)(1-\omega)}\right) > \left(\frac{\theta - \omega L}{(1-\lambda)(1-\omega)}\right) u'\left(\frac{\theta - \omega L}{(1-\lambda)(1-\omega)}\right) \quad (16)$$

This means,

$$u\left(\frac{\theta - \omega L}{(1-\lambda)(1-\omega)}\right) > \left(\frac{\theta - L}{(1-\lambda)(1-\omega)}\right) u'\left(\frac{\theta - \omega L}{(1-\lambda)(1-\omega)}\right) \quad (17)$$

It is clear that $\frac{dEU}{de} < 0$. Therefore, noise e should be at a minimum in order to maximize expected utility of the depositor.

3. CONCLUSION

When depositors receive noisy signals about the future returns of a bank, the probability of bank runs and their expected returns depend on the noise. The level of noise differs depending on bank policies, bank regulators' policies, education level of agents etc. If the probability of a partial run is big enough, noise level should be taken into consideration in the analysis. This model recommends that in order to maximize the expected utility of the depositors and to minimize the probability of bank runs, depositors should be given accurate information about the banks' future profitability in a manner they can understand and interpret the information that is available.

REFERENCES

Carlsson, H., van Damme, E. (1993). `Global Games and Equilibrium Selection', *Econometrica*, Vol 61, No. 5, pp. 989 - 1018.

Dasgupta, A. (2004). `Financial Contagion Through Capital Connections: A Model of the Origin and Spread of Bank Panics', *Journal of European Economics Association*, pp. 1049 - 1084.

Diamond, D.W., Dybvig, P. H. (1983). `Bank Runs, Deposit Insurance and Liquidity', *Journal of Political Economy* 91, pp. 401 - 419.

Erland, W. N. (2005). `Bank Stability and Transparency', *Journal of Financial Stability*, pp. 342 - 354.

Goldstein, I.., Pauzner, A. (2004). `Contagion of Self-fulfilling Financial Crisis Due to Diversification of Investment Portfolios', *Journal of Economic Theory*, pp. 151 - 183.

Goldstein, I.., Pauzner, A. (2005). `Demand Deposit Contracts and the Probability of Bank Runs', *Journal of Finance*, pp. 1293 - 1327.

Morris, S., Shin, H. S. (2002). `Global Games: Theory and Applications', Advances in Economics and Econometrics, the Eighth World Congress. Edited by Cambridge University Press.

Nier, E.W. (2005). `Bank Stability and Transparency', *Journal of Financial Stability*, pp. 342 - 354.